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Simplectic Geometry and the Canonical Variables for Dirac–Nambu–Goto and Gauss–Bonnet System in String Theory

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Using a strongly covariant formalism given by Carter for the deformations dynamics of p-branes in a curved background and a covariant and gauge invariant geometric structure constructed on the corresponding Witten's phase space, we identify the canonical variables for Dirac–Nambu–Goto (DNG) and Gauss–Bonnet (GB) system in string theory. Future extensions of the present results are outlined.

KEY WORDS: symplectic geometry; string theory; quantization.

1. INTRODUCTION

The interest in physical systems characterized by extended structures goes back to the 19th century and to Lord Kelvin's "aether atoms," for which a spatial extension was postulated in order to accommodate a complex structure which would behave both as an elastic solid (conveying the transverse wave motion of electromagnetism) and viscous liquid (dragged by the earth in its orbital motion).

In the 20th century, there have been three active motivations leading to either classical or quantum extendons. On the other hand, the physic of condensed matter (including biological systems) have revelated that membranes and two-dimensional layers play an important role; in some case, there also appear one-dimensional filaments (or strings). Similar structures appear in astrophysics and cosmology, one example being the physics of black holes, in which the "membrane" is the boundary layer between the hole and the embedding spacetime, and another example is represented by the hypothetical cosmic strings.

In last years a considerable amount of effort has been devoted for developing a quantum field theory of such extended objects (which in fact, will constitute

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the ultimate framework) for a complete M theory; however, it has not yet been fully developed. The problem is that the dynamics of extended objects is highly nonlinear and the standard methods are not directly applied. However, using a covariant canonical formalism introduced by Crncović and Witten (1987) in recent letters (Cartas-Fuentevilla, 2002; Cartas-Fuentevilla and Escalante, 2004; Escalante, 2004a,b) the basic elements to quantize extended objects (in particular bosonic p-branes) have been explored, for example, in Escalante (2004a) we established the bases to study the quantization aspects of p-branes with thickness, because, when adding it to the DNG action has an important effect on QCD (Polyakov, 1986; Kleinert, 1986), among other things. In Escalante (2004b), it has been demonstrated that the presence of Gauss–Bonnet (GB) topological term in the DNG action describing strings, has a dramatic effect on the covariant phase space formulation of the theory, in this manner, we shall obtain a completely different quantum field theory. Recently, using the results given in Escalante (2004b) we identified the covariant canonical variables for DNG p-branes and GB strings, among other things Escalante (2004c). However, we found a little problem, because, the canonical variables found for DNG are identified with spacetime indices, whereas, the canonical variables for GB strings with worldsheet indices, in this manner, if we add the GB term to the DNG action describing strings, we need identify the pullback on the canonical variables for GB strings in order to obtain it in terms of spacetime indices, and thus, to study in a covariant form the quantization aspects for DNG–GB system in string theory, but it was not clarified.

In this manner, the purpose of this article is to make, first, a generalization of the results presented in Escalante (2004a,b) for a general Lagrangian constructed locally from the geometry of the worldvolume in an arbitrary background, after that, using a strongly covariant formalism given by Carter (1997) and Carter (1993), we identified the canonical variables for DNG–GB system in string theory, in this manner, we resolved the problem found in Escalante (2004c).

This paper is organized as follows. In Section 2, we make a generalization of the method utilized in Escalante (2004a,b) for a Lagrangian constructed from the geometry of the worldvolume embedding in an arbitrary background, confirming as special case the results found in Escalante (2004a). In Section 3, we make an outline of the results found in Cartas-Fuentevilla (2003), which, will be important for the developement of this paper. In Section 4, using a strongly covariant scheme of deformations introduced by Carter (1997) and Carter (1993), we found the canonical variables for GB system in string theory, that unlike (Escalante, 2004c), the canonical variables of GB have spacetime indices, which will be determinant for the treatment of DNG–GB strings. In Section 5, with the results found in the previous sections we identified the canonical variables for DNG–GB system in string theory, and with this result, we clarified the problem that we found in Escalante (2004c). In Section 6, we give the conclusions and prospects.

2. SYMPLECTIC POTENTIALS FOR P-BRANES IN A CURVED BACKGROUND

In recent letters, a covariant and gauge invariant symplectic structure for DNG p-branes (Cartas-Fuentevilla, 2002), for membranes with quadratic terms in the extrinsic curvature (Escalante, 2004a), and for the Gauss–Bonnet topological term propagating in a curved background (Escalante, 2004b) has been constructed. The form of constructing this geometric structure is by means of identifying from the arguments of the total divergences at the level of the Lagrangian a symplectic potential that does not contribute locally to the dynamics of the system, but its variation (the exterior derivative on the phase space) generates a geometric structure. In this manner, the purpose of this section is to generalize these results for a Lagrangian constructed from the geometry of the worldvolume embedding in an arbitrary spacetime.

For our aims, we will consider a local action depending on the embedding functions X^μ which is invariant both under worldvolume reparametrization and under rotations of the normals given by

$$S[X] = \int \sqrt{-\gamma} L d^D \xi, \tag{1}$$

where the Lagrangian L will be constructed locally from the geometry of the worldvolume as

$$L\big(\gamma^{ab}, K^i_{ab}, \widetilde{\nabla}_a K^i_{bc}\big), \tag{2}$$

here, γ^{ab}, K^i_{ab} and $\widetilde{\nabla}_a$ is the metric induced, the extrinsic curvature and the covariant derivative under rotation of the normal vector field, respectively (Capovilla and Guven, 1995).

Now, we need calculate the deformation of the Lagrangian given in Equation (2) to identify the equations of motion and the symplectic potential for the theory described by the action (1). For this, we decompose an arbitrary infinitesimal deformation of the embedding δX^μ into its parts tangential and normal to the worldvolume, this is

$$\delta X^\mu = e^\mu{}_a \phi^a + n^\mu{}_i \phi^i, \tag{3}$$

where n^μ_i are the vector fields normal and e^μ_a are the vector fields tangent to worldvolume, thus, the deformation operator is defined as

$$D = D_\delta + D_\Delta, \tag{4}$$

where

$$D_\delta = \delta^\mu D_\mu, \qquad \delta^\mu = n_i{}^\mu \phi^i, \tag{5}$$

and

$$D_{\Delta} = \Delta^{\mu} D_{\mu}, \qquad \Delta^{\mu} = e_a{}^{\mu}\phi^a, \tag{6}$$

in this manner, the variation of Equation (1) with the Lagrangian (2) is given by

$$\delta S = \int \sqrt{-\gamma}\nabla_a(L\phi^a)d^D\xi + \int \sqrt{-\gamma}\big[K^i\phi_i L + H_i^{ab}\widetilde{D}_\delta K^i_{ab} + H^{ab}D_\delta\gamma_{ab} + H_i^{abc}\widetilde{D}_\delta\big(\widetilde{\nabla}_a K^i_{bc}\big)\big]d^D\xi, \tag{7}$$

where

$$H_{ab} = \frac{\partial L}{\partial\gamma_{ab}}, \quad H_i^{ab} = \frac{\partial L}{\partial K^i_{ab}} = H^{ba}{}_i, \quad H_i^{abc} = \frac{\partial L}{\partial\widetilde{\nabla}_a K^i_{bc}} = H_i^{acb}. \tag{8}$$

On the other hand, using the deformation formalism introduced in Capovilla and Guven (1995) and writing the normal variation of γ_{ab}, K^i_{ab} and $\widetilde{\nabla}_a K^i_{cb}$ in a curved background, we obtain

$$\widetilde{D}_\delta K^i_{ab} = -\widetilde{\nabla}_a\widetilde{\nabla}_b\phi^i + K^i_{ac}K^c_{bj}\phi j + g(R(e_a, n_j)e_b, n^i), \tag{9}$$

$$\begin{aligned}\widetilde{D}_\delta\widetilde{\nabla}_a K^i_{bc} = {}& \widetilde{\nabla}_a\big[-\widetilde{\nabla}_b\widetilde{\nabla}_c\phi^i + K^i_{db}K^d_{cj}\phi^j + g(R(e_b, n_j)e_c, n^i)\phi^j\big] - \big[\widetilde{\nabla}_b\big(K_a^{gj}\phi_j\big) \\ & + \widetilde{\nabla}_a\big(K_b^{gj}\phi_j\big) - \widetilde{\nabla}^g\big(K^j_{ba}\phi_j\big)\big]K^i_{gc} + \big[-\widetilde{\nabla}_c\big(K_a^{gj}\phi_j\big) - \widetilde{\nabla}_a\big(K_c^{gj}\phi_j\big) \\ & + \widetilde{\nabla}^g\big(K^j_{ca}\phi_j\big)\big]K^i_{gb} + \big[K^i_{ad}\widetilde{\nabla}^d\phi^j - K^j_{ad}\widetilde{\nabla}^d\phi^i \\ & - g(R(n_k, e_a)n^j, n^i)\phi^k\big]K_{bcj},\end{aligned} \tag{10}$$

with $g(R(e_a, n_j)e^a, n^i) = R_{\alpha\beta\mu\nu}n_j{}^{\alpha}e_a{}^{\beta}e^{a\mu}n^{i\,\nu}$, $R_{\alpha\beta\mu\nu}$ being the background Riemann tensor (Capovilla and Guven, 1995; Escalante, 2004a).

Substituting Equations (9) and (10) and removing the scalar field ϕ^i in (7) we obtain

$$\begin{aligned}\delta S = \int \sqrt{-\gamma}\big[&K^i L - 2K^{abi}H_{ab} - \widetilde{\nabla}_a\widetilde{\nabla}_b H_i^{ab} - K^i_{ac}K_b^{cj}H_j^{ab} \\ & + g(R(e_a, n^i)e_b, n^j))H_j^{ab} + \widetilde{\nabla}_c\widetilde{\nabla}_b\widetilde{\nabla}_a H^{abci} + 2K_a^{gi}\widetilde{\nabla}_b\big(H_j^{abc}K_{gc\,j}\big) \\ & + 2K_b^{gi}\widetilde{\nabla}_a\big(H_j^{abc}K_{gc\,j}\big) + K^i_{ba}\widetilde{\nabla}^g\big(H_j^{abc}\big)K^j_{gc} + K_a{}^{gi}\widetilde{\nabla}_a\big(H_j^{abc}K^j_{gb}\big) \\ & - K^i_{ca}\widetilde{\nabla}^g\big(H_j^{abc}K^j_{gb}\big) - \widetilde{\nabla}^d\big(H_j^{abc}K^j_{ad}K^i_{bc}\big) - \widetilde{\nabla}^d\big(H^{abci}K^j_{ad}K_{bcj}\big)\end{aligned}$$

$$
\begin{aligned}
&-g(R(n^i, e_a)n^j, n^l)H_l^{abc}K_{bcj}\big]\phi_i \\
&+\int \sqrt{-\gamma}\widetilde{\nabla}_a\big[L\phi^a - H_i^{ab}\widetilde{\nabla}_b\phi^i + \widetilde{\nabla}_b H_i^{ab}\phi^i - H_i^{abc}\widetilde{\nabla}_b\widetilde{\nabla}_c\phi^i \\
&+H_i^{abc}g(R(e_b, n_j)e_c, n^i)\phi^j + H_i^{abc}K_{db}^iK_{cj}^d\phi^j \\
&+\widetilde{\nabla}_b H_i^{abc}\widetilde{\nabla}_c\phi^i - \widetilde{\nabla}_b\widetilde{\nabla}_c H_i^{cba}\phi^i - H_i^{bac}K_{gc}^iH_b^{gj}\phi_j - 2H_i^{abc}K_{gc}^iK_b^{gj}\phi_j \\
&-H_i^{cba}K_{gb}^iK_c^{gj}\phi_j + H_i^{gbc}K_c^{ai}K_{bg}^j\phi_j + H_i^{gbc}K_b^{ai}K_{cg}^j\phi_j \\
&+H_i^{gbc}K_{bcj}K_g^{ai}\phi^j - H_i^{dbc}K_d^{aj}K_{bcj}\phi^i\big]d^D\xi, \qquad (11)
\end{aligned}
$$

from the last equation we can identify the equations of motion given by

$$
\begin{aligned}
&K^iL - 2K^{ab^i}H_{ab} - \widetilde{\nabla}_a\widetilde{\nabla}_b H^{ab^i} - K_{ac}^iK_b^{cj}H_j^{ab} + g(R(e_a, n^i)e_b, n^j))H_j^{ab} \\
&+ \widetilde{\nabla}_c\widetilde{\nabla}_b\widetilde{\nabla}_a H^{abci} + 2K_a^{gi}\widetilde{\nabla}_b\big(H_j^{abc}K_{gc\,j}\big) + 2K_b^{gi}\widetilde{\nabla}_a\big(H_j^{abc}K_{gc\,j}\big) \\
&+ K_{ba}^i\widetilde{\nabla}^g\big(H_j^{abc}\big)K_{gc}^j + K_a{}^{gi}\widetilde{\nabla}_a\big(H_j^{abc}K_{gb}^j\big) - K_xca^i\widetilde{\nabla}^g\big(H_j^{abc}K_{gb}^j\big) \\
&- \widetilde{\nabla}^d\big(H_j^{abc}K_{ad}^jK_{bc}^i\big) - \widetilde{\nabla}^d\big(H^{abci}K_{ad}^jK_{bcj}\big) \\
&- g(R(n^i, e_a)n^j, n^l)H_l^{abc}K_{bcj} = 0, \qquad (12)
\end{aligned}
$$

and we identify from the pure divergence term in (11)

$$
\begin{aligned}
\Psi^a = \sqrt{-\gamma}\big[&L\phi^a - H_i^{ab}\widetilde{\nabla}_b\phi^i + \widetilde{\nabla}_b H_i^{ab}\phi^i - H_i^{abc}\widetilde{\nabla}_b\widetilde{\nabla}_c\phi^i \\
&+H_i^{abc}g(R(e_b, n_j)e_c, n^i)\phi^j + \widetilde{\nabla}_b H_i^{abc}\widetilde{\nabla}_c\phi^i - \widetilde{\nabla}_b\widetilde{\nabla}_c H_i^{cba}\phi^i \\
&-H_i^{bac}K_{gc}^iK_b^{gj}\phi_j - H_i^{abc}K_{gc}^iK_b^{gj}\phi_j - H_i^{cba}K_{gb}^iK_c^{gj}\phi_j \\
&+2H_i^{gbc}K_c^{ai}K_{bg}^j\phi_j + H_i^{gbc}K_{bcj}K_g^{ai}\phi^j - H_i^{dbc}K_d^{aj}K_{bcj}\phi^i\big], \qquad (13)
\end{aligned}
$$

as a symplectic potential for the theory described for a Lagrangian given in Equation (2), which is ignored in the literature, as it does not contribute locally to the dynamics, but generates our geometrical structure on the phase space. Note that there exists a term involving explicitly the background curvature in Equation (13).

Now, next we will take particular cases of the Lagrangian given in Equation (2). Using the previous results we will confirm the results given in Escalante (2004a); for this, we take as first example the DNG p-branes action.

As we know the DNG p-branes action is proportional to the area of the spacetime trajectory created by the brane, thus, if we take to $L = -\mu$, where μ is a constant characterizing the brane tension we have the well-known action for

DNG p-branes

$$S = -\mu \int \sqrt{-\gamma} d^D \xi, \tag{14}$$

in this manner, utilizing Equation (8) we easily obtain

$$\begin{aligned} H_{ab} &= 0, \\ H_i^{ab} &= 0, \\ H_i^{abc} &= 0, \end{aligned} \tag{15}$$

substituting the last result in Equation (12) we obtain

$$K^i = 0, \tag{16}$$

that corresponds to the equations of motion for DNG p-branes describing extremal surfaces (Capovilla and Guven, 1995; Cartas-Fuentevilla, 2002; Escalante, 2004a).

On the other hand, if we consider Equation (15) in (13) we find

$$\Psi^a = -\mu \sqrt{-\gamma} \phi^a, \tag{17}$$

that corresponds to the symplectic potential for DNG p-branes. Thus, if we take the variation of Ψ^a (the exterior derivative on the phase space) given in (17) we will generate a geometrical structure on the phase space, for more details see Escalante (2004a).

As second example we will consider a Lagrangian that is quadratic in the extrinsic curvature, because of in many cases it was seen that DNG action is inadequate and there are missing corrective quadratic terms in the extrinsic curvature. For example, in the 1980s Polyakov proposed a modification to the DNG action by adding a rigidity term constructed with the extrinsic curvature of the worldsheet generated by a string, and to include quadratic terms in the extrinsic curvature to the DNG action is absolutely necessary, because its influence on the infrared region determines the phase structure of the string theory, in this manner, we can compute the critical behavior of random surfaces and their geometrical and physical characteristics (Kleinert, 1986; Polyakov, 1986). In the treatment of topological defects (Maeda and Turok, 1988), curvature terms are induced by considering an expansion in the thickness of the defect. Bosseau and Letelier have studied cosmic strings with arbitrary curvature corrections, finding for example, that the curvature correction may change the relation between the string energy density and the tension (Bosseau and Lettelier, 1992). Furthermore, such models have been used to describe the mechanical properties of lipid membranes (Canham, 1970; Helfrich, 1973). Because of the above considerations, we will take a Lagrangian quadratic in the extrinsic curvature given by $L = \alpha K^i K_i$, here

α is a constant associated with the brane tension (Capovilla and Guven, 1995; Escalante, 2004a). Thus, if we substitute it in Equation (8) we obtain

$$\begin{aligned} H_{ab} &= 2\alpha K^i K_{abi}, \\ H_i^{ab} &= 2\alpha \gamma^{ab} K_i, \\ H_i^{abc} &= 0. \end{aligned} \tag{18}$$

In this manner, in virtue to last equation Equation (8) takes the form

$$\widetilde{\Delta} K^i + \left(-g(R(e_a, n^j)e^a, n^i) + \left(\gamma^{ac}\gamma^{bd} - \frac{1}{2}\gamma^{ab}\gamma^{cd} \right) K_{ab}^j K_{cd}^i \right) K_j = 0, \tag{19}$$

that corresponds to the dynamics for the theory under study (Capovilla and Guven, 1995; Escalante, 2004a).

In the same form, if we substitute Equation (18) into (13) we obtain

$$\Psi^a = 2\alpha\sqrt{-\gamma} \left[\frac{1}{2} K^j K_j \phi^a + \phi_i \widetilde{\nabla}^a K^i - K_i \widetilde{\nabla}^a \phi^i \right], \tag{20}$$

that corresponds to the integral kernel of a covariant and invariant of gauge symplectic structure defined on the covariant phase space (Escalante, 2004a).

In concluding this section it is important to mention that in the same form, using the previous results we can obtain the results presented in Escalante (2004b); in this case, we analyze what happens when we add the GB topological term to the DNG action in string theory, and we found for example, that in the dynamics of deformations exist a non trivial contribution because of the GB topological term, therefore, we found a contribution that does not vanish in the symplectic structure constructed on the covariant phase space for the DNG–GB system in string theory. These important results allowed us to find using a weakly covariant formalism Capovilla and Guven (1995), the canonical variables for DNG p-branes and GB term in string theory Escalante (2004c), however, as we already commented we found some problems in considering the DNG–GB complete system. In this manner, in the next section we will use a strongly covariant formalism introduced in Carter (1993, 1997) and the results presented in Cartas-Fuentevilla (2003) for this problem can be clarified, in other words, we will find the canonical variables for DNG–GB system in string theory, which is completely unknown in the literature.

3. THE CANONICAL VARIABLES FOR DNG SYSTEM

As we commented, in Escalante (2004c) we found the canonical variables for DNG p-branes (that contain the particular case of strings theory) using a weakly covariant formalism Capovilla and Guven (1995), with spacetime indices, and the canonical variables for GB topological term with worldsheet indices. In this manner, if we consider the DNG–GB system we need rewrite the canonical variables of GB term with background spacetime indices and to consider a canonical transformation that leaves the symplectic structure in the Darboux form with some new variables, say P and Q. However, this problem can be clarified using a strongly covariant formalism introduced in Carter (1997) and Carter (1993) as we will see in the next lines.

Using a strongly covariant formalism introduced by Carter (1997) and Carter (1993), it is found that the symplectic structure for DNG branes in a curved background is given by Cartas-Fuentevilla (2003)

$$\omega = \sigma_0 \int_\Sigma \delta(-\sqrt{-\gamma}\eta^\mu{}_\alpha \xi^\alpha)\, d\bar{\Sigma}_\mu = \int_\Sigma \sqrt{-\gamma}\,\widetilde{J}^\mu d\bar{\Sigma}_\mu, \tag{21}$$

where σ_0 is a fixed parameter, $\eta^\mu{}_\alpha$ is the (first) fundamental tensor, $\sqrt{-\gamma}\,\widetilde{J}^\mu = \delta(-\sigma_0\sqrt{-\gamma}\eta^\mu{}_\alpha\xi^\alpha)$, Σ being a (spacelike) Cauchy surface for the configuration of the brane, while $d\bar{\Sigma}_\mu$ is the surface measure element of Σ, and is normal to Σ and tangent to the world surface. Here δ is identified as an exterior derivative on the covariant phase space. The symplectic structure given in (21) is a exact differential form, as it comes from the exterior derivative of a one form and in particular is an identically closed two-form on the phase space. The closeness is equivalent to the Jacoby identity that Poisson brakets satisfy, in a usual Hamiltonian scheme, and the symplectic current is (world surface) covariantly conserved ($\bar{\nabla}_\mu \widetilde{J}^\mu = 0$), which guarantees that ω is independent on the choice of Σ and, in particular, is Poncaré invariant.

We can rewrite the symplectic structure given in (25) for identifying the canonical variables for DNG branes in the next form

$$\omega = \int_\Sigma \delta X^\alpha \delta \hat{p}_\alpha d\Sigma, \tag{22}$$

where $\hat{p_\alpha} = \sqrt{-\gamma}\,p_\alpha$, and $p_\alpha = \sigma_0\tau_\alpha$, τ_α being a unit timelike vector field. In this manner, Eq. (32) allows us to identify X^μ and $\hat{p_\alpha}$ as the canonical conjugate variables in this covariant description of the phase space for DNG branes in a curved background (in Escalante (2004c) we identified the canonical variables for DNG p-branes in a weakly covariant formalism). It is important to mention that the symplectic structure given in Equation (21) and the identification of the canonical variables X^μ and $\hat{p_\alpha}$ allows us to find, for example, the covariant Poisson brackets, the Poncaré charges and the closeness of the Poincaré algebra (Cartas-Fuentevilla, 2003; Escalante, 2004c).

4. THE CANONICAL VARIABLES FOR GB SYSTEM IN STRING THEORY

As we know, the Einstein–Hilbert term is characterized for the action

$$S = \sigma_1 \int \sqrt{-\gamma} R d\bar{\Sigma}, \tag{23}$$

where σ_1 is a fixed parameter and R is the scalar curvature of the embedding Cartas-Fuentevilla and Escalante (2004). Using the deformations formalism given in Carter (1997) and Carter (1993) we can calculate the variation of S obtaining

$$\delta S = 2\sigma_1 \int \sqrt{-\gamma} G^{\gamma\nu} K_{\gamma\nu\mu} \xi^\mu d\bar{\Sigma} + \sigma_1 \int \sqrt{-\gamma} \bar{\nabla}_\mu \big(-2G^\mu_\nu \xi^\nu + \eta^{\alpha\beta} \delta \rho_{\alpha}{}^{\mu}_{\beta} - \eta^\alpha_\beta \eta^{\mu\tau} \delta \rho_{\alpha}{}^{\beta}_{\tau} \big) d\bar{\Sigma}, \tag{24}$$

where $G^{\gamma\nu}$ is the internal adjusted Ricci tensor, $K_{\gamma\nu\mu}$ is the second fundamental tensor and $\rho^\mu_{\alpha\beta}$ is the frame gauge internal rotation pseudo-tensor or internal connection (Carter, 1993, 1997). In general, the adjusted Ricci tensor does not vanish for an imbedded p-surface, however, in string theory the Ricci tensor vanishes identically. From the last equation we can identify the equations of motion for the brane theory given by

$$G^{\gamma\nu} K^\mu_{\gamma\nu} = 0, \tag{25}$$

and as in Section 2, the total divergence term of Equation (24) is identified as symplectic potential for the theory under study, given by

$$\Psi^\mu = \sigma_1 \sqrt{-\gamma} \big[-2G^\mu_\nu \xi^\nu + \eta^{\alpha\beta} \delta \rho^\mu_{\alpha\beta} - \eta^\alpha_\beta \eta^{\mu\tau} \delta \rho^\beta_{\alpha\tau} \big]. \tag{26}$$

If we take the particular case of string theory in Equation (25) the adjusted Ricci tensor vanishes, in this manner, if we utilize the standard canonical formalism to quantize this system, we would not find apparently nothing interesting, however, as we can see in Escalante (2004b) using a weakly covariant formalism introduced in Capovilla and Guven (1995) we found that the GB term in string theory gives a nontrivial contribution on the Witten covariant phase space leading to a completely different quantum field theory. We can see it if we take the particular case of string theory in Eq. (26) obtaining

$$\Psi^\mu = \sigma_1 \sqrt{-\gamma} \big[\eta^{\alpha\beta} \delta \rho^\mu_{\alpha\beta} - \eta^\alpha_\beta \eta^{\mu\tau} \delta \rho^\beta_{\alpha\tau} \big], \tag{27}$$

in this manner, we can see that the terms of last equation do not vanish. This result allows us to find the canonical variables for GB strings.

In order to continue, we need rewrite the internal connection in terms of the (co) vector ρ_μ defined as

$$\rho_\lambda = \rho_{\lambda}{}^{\mu}_{\nu} \varepsilon^\nu_\mu, \quad \rho_{\lambda}{}^{\mu}_{\nu} = \frac{1}{2} \varepsilon^\mu_\nu \rho_\lambda, \tag{28}$$

where $\varepsilon^{\mu\nu} = 2\iota_0^{[\mu}\iota_1^{\nu]}$, ι_0^μ being a time-like unit vector, and ι_1^μ a space-like one, which constitute an orthonormal tangent (to the world sheet) frame (Carter, 1993, 1997). Thus, considering the last equation, the symplectic potential given in the expression (27) takes the form

$$\Psi^\mu = \sqrt{-\gamma}\varepsilon^{\mu\nu}\delta\rho_\nu, \tag{29}$$

where we have used the frame gauge property of $\rho_{\lambda}{}^\mu_\nu$ and consequently of ρ_λ (Cartas-Fuentevilla and Escalante, 2004).

In this manner, we can define a covariant and gauge invariant symplectic structure for GB strings as

$$\omega' = \int_\sigma \delta(\sigma_1\sqrt{-\gamma}\varepsilon^{\mu\nu}\delta\rho_\nu)\, d\bar{\Sigma}_\mu, \tag{30}$$

therefore, from the last equation we can identify as well as for DNG system the canonical variables for GB strings, this is

$$p_\nu = \sigma_1\sqrt{-\gamma}\varepsilon^\mu{}_\nu\tau_\mu, \quad q^\nu = \rho^\nu. \tag{31}$$

In this manner, we can see that in this case the canonical variables have spacetime indices contrary to Escalante (2004c) that has worldsheet indices. With these results we can treat the complete DNG–GB system which is the purpose of the next section and this paper.

5. THE CANONICAL VARIABLES FOR DNG–GB SYSTEM IN STRING THEORY

In this section, we will study the DNG–GB system in string theory. For that, we begin with the action that describe the system under study, this is

$$S = -\sigma_0 \int \sqrt{-\gamma} d\Sigma + \int \sigma_1\sqrt{-\gamma} R d\Sigma, \tag{32}$$

now, using the deformations formalism given in Carter (1997) and Carter (1993) we take the variation of last equation and considering the particular case of string theory, finding

$$\delta S = \sigma_0 \int \sqrt{-\gamma} K_\mu \xi^\mu d\Sigma + \int \bar{\nabla}_\mu[-\sigma_0\eta^\mu{}_\nu\xi^\nu + \sigma_1\varepsilon^{\mu\nu}\delta\rho_\nu] d\Sigma, \tag{33}$$

where we can identify the equations of motion given by

$$K^\mu = 0, \tag{34}$$

that corresponds to the equations of motion for DNG strings. On the other hand, the total divergence term of Eq. (33) is identified as symplectic potential for the theory under study

$$\Psi^{\mu} = \sqrt{-\gamma}[-\sigma_0 \eta^{\mu}{}_{\nu} \xi^{\nu} + \sigma_1 \varepsilon^{\mu\nu} \delta\rho_{\nu}], \tag{35}$$

With the previous results, from last equation we can obtain the covariant and gauge invariant symplectic structure for DNG–GB in string theory, this is

$$\omega = \int_{\Sigma} \delta \hat{P}_{\nu} \wedge \delta Q^{\nu} d\bar{\Sigma}, \tag{36}$$

where

$$\hat{P}_{\nu} = \sqrt{-\gamma}\, p_{\nu}, \quad \text{and} \quad Q^{\nu} = -\frac{\sigma_1}{\sigma_0} \varepsilon^{\nu\alpha} \rho_{\alpha} + X^{\nu}, \tag{37}$$

with $p_{\nu} = \sigma_0 \tau_{\nu}$. Therefore, we can identify $\hat{P}_{\nu}$ and Q^{ν} as canonical variables for DNG–GB system in string theory which is completely unknown in the literature. We can note that the contribution because of GB term on the canonical variable Q^{ν} (see the first term of Q^{ν} in Equation (37)) will be relevant when is calculated the angular momentum of the complete DNG–GB system, and will be important in the complete quantum field theory.

It is important to mention that we have choose the canonical momentum for DNG strings (see Equation (22)) and DNG–GB strings (see Equation (37)) in the same form, the reason is that p_{ν} satisfies the mass shell ($p_{\nu} p^{\nu} = \sigma^2$) and as we know of the literature the mass shell is an important condition to quantum level for DNG strings, because of the Virasoro operators and the mass shell conditions determine the masses of the physical states, in this manner, we also hope that such condition will be important when we analyze the spectrum of DNG–GB system in string theory, however this we discuss in future works.

In concluding this work, is important to see that if we take $\sigma_1 = 0$ in Equation (37) we obtain the result given in Eq. (22). However, we hope the choice of the canonical variables made in this paper as first quantization are the best election, because the canonical momentum for DNG and DNG–GB strings coincide, thus, with the results of this paper and the treatment that is found in the literature to quantize DNG strings we have the necessary elements to quantize DNG–GB strings.

6. CONCLUSIONS AND PROSPECTS

As we can see, using the deformations formalism introduced by Carter and a covariant and gauge invariant symplectic structure, we could find the canonical variables for DNG–GB system in string theory which is absent in the literature. With these results we have the necessary elements to study the quantization aspects of DNG–GB strings, as for this purpose we need the results of this paper and the solutions to the equation of motion (34) that are given in elementary books on string theory. In this manner, we can observe the change in the resulting quantum field theory of the topology of the world surface given by GB term, and thus, find the contribution of such term to the results that we find in the literature for DNG strings; however, we will discuss this subject in future works.

In addition to this work, we know that the bosonic strings (which is the case of this work) are not the general case to describe the nature and it is necessary to add the supersymmetry, among other things, in order to give a description of the fermionic matter. In this manner, a interesting question may be the inclusion of supersymmetry to the results of this paper to find the quantization bases for DNG–GB in superstring theory and thus giving a complete description of the matter, however, we will discuss this in future works.

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